Multi choice knapsack problem, Case Study in Optimal Television Adverts Selection

Assistant ProfessorDr.E.V.Ivokhin1,
Assistant Lecturer
AlmodarsBarraq.Subhi Kaml2
1Taras Shevchenko National University of Kyiv,
E-mail: ivohin@univ.kiev.ua;
[2]*Taras Shevchenko National University of Kyiv,*

E-mail: *barraq_1978@ukr.net*

الملخص

تعتبر مسائل الحقيبة من بين ابسط مسائل البرمجة العددية اذ ان المسائل من هذا النوع ترتبط عادةً باختيار مواد من مجموعة معطاه وبمحدودية لقيمة المادة وحجمها او وزنها ، على ان لا يزيد مجموع تلك الاوزان عن حد ما، في هذا البحث سنتناول تطبيقات مسائل الحقيبة التقليدية الثنائية بأربع قيود لاختيار امثل لعدد من الاعلانات من بين مجموعة خلال فترات الذروة لتحقيق اعلى الايرادات، بعد ان تم بناء نموذج رياضي للبيانات المتوفرة وباستخدام برنامج تطبيقات الجاهزة تم التوصل الى الحل الامثل . اما الجزء الثاني من البحث فقد تم التطرق الى الحل لما بعد الامثلية او ما يسمى التحليل الحساس للمسئلة قيد الدراسة .

Abstract

The Knapsack Problems are among the simplest integer programs. Problems in this class are typically concerned with selecting from a set of given items, each with a specified weight and value, a subset of items whose weight sum does not exceed a prescribed capacity and whose value is maximum.

In this research paper, we shall consider the application of classical $0-1$ knapsack problem with four constraints to selection of television advertisements at critical periods such as prime time news, news adjacencies, Break in News and peak times using the program WINQSB. In the second part of this paper we treated with solution post optimality (sensitivity analysis) by using principles of shadow prices and reduce cost in the model above since real problems are often degenerate, the output from conventional LP software regarding such marginal information can be misleading. This paper surveys and generalizes known results in this topic and demonstrates how true shadow prices and reduce cost can be computed with or without modification to existing software [2].

Key words: Advertisements, integer programming Knapsack, Shadow prices and Reduce costs.

1.Introduction

Knapsack problems have been intensively studies since the pioneering work of Dantzing [3] in the late 50's, both because of their immediate applications in industry and financial management, but more pronounced for theoretical reasons, as Knapsack problems frequently occur by relaxation of various integer programming problems. In such applications, we need to solve a Knapsack problems each time a bounding function is derived demanding extremely fast solution techniques. The family of Knapsack problems all require a subset of some given items to chosen such that the corresponding profit sum is maximizing without exceeding the capacity of the knapsack(s). In the 0-1 Knapsack problems each item may be chosen at most once, while. The multi-choice Knapsack problems occur when the items should be chosen from disjoint classes. Sinha and Zoltners [4] proposed to use multi-choice Knapsack problems to select which components should be linked in series in order to maximizing fault tolerance.

Moreover Nauss[5] proposed to transform nonlinear knapsack problems to multi-choice Knapsack problems. In the second category we should mention that the 0-1 Knapsack problem appears as sub problem when solving generalized assignment problem, which again is heavily used when solving Vehicle Routing Problem ,G.Laporte[6].

2.Knapsack problems and analysis data

Suppose the producer of a TV programme wants to select among numerous adverts for the prime time (news at 19:00 h GMT), which is interspersed with five or six spots of adverts of not more than three minutes each. It is self-evident that the optimal solution of the knapsack problem above will indicate the best possible choice of investment [1].

The objects to be considered will generally be called items and their number by n. The value and size associated with the j th item will be called profit (cost of Advert) and weight (duration of advert) respectively.

The traditional 0−1 Knapsack Problem (KP) for this case can be mathematically formulated through the following integer linear programming

$$\sum_{j=1}^{n} v_j x_j \to Max \qquad (2.1)$$

subject to

$$\sum_{j=1}^{n} w_j x_j \le W \quad , x_j \in \{0,1\}, j = \overline{1,n} \qquad (2.2)$$

where v_j is value (cost of advert) and w_j is the weight (duration of advert) of the *jth* item respectively, , $j = \overline{1,n}$, and W is the maximum time allocated for adverts.

If we know quantity of different adverts categories, costs and weights in each category, the model above can be rewrite as:

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} v^i{}_j x_{ji} \to Max$$

$$\sum_{j=1}^{n_i} w^i{}_j x_{ji} \le W^i, x_{ji} \in \{0,1\}, i = \overline{1,m}$$

Where (m) represented the quantity of categories, n_i, $i = \overline{1,m}$ categories of advertise in every category .

There are two basic methods for solving the 0−1 knapsack problems (KP): Theses are Branch-and-Bound techniques have frequently been applied to Knapsack problems since Kolesar [7], and dynamic programming methods. However we uses the first method in software WINQSB to have been used to solve large scale problems. This study was undertaken using data collected fromS.K.Amponsah [1].

TV is a public broadcaster which depends to the greater extent on government subvention. Broadcasting Corporation is however mandated to generate revenue to supplement the government subvention. To this end TV has various ways of generating additional income. These include sponsorship of programmes, social and funeral announcements, advertisements among others. However, this research focused on advertisements, which are slotted in the programmes schedules prepared quarterly to generate additional income to sustain the operations of the TV station. TV uses an arbitrary method in the selection. In this process an advert is accepted if there is an available time without regard to optimizing revenue. The category of adverts selection studied included:

- Prime time news (19.00 h GMT)
- News adjacencies (five minutes before and after

news at 12.00, 14.00, 19.00 and 22.30 h GMT)

- Other News time (12.00, 14.00, 19.00, 22.30 h GMT)
- Break in programmes (peak and off peak)

Table 1 shows the various rates for the different categories of adverts at TV. For example Prime time News adverts for 15 sec costs $215 while for 60 sec, the rate is $750. The rates are high for Prime time News and news adjacencies. These are periods where most customers want their adverts televised to reach a larger TV audience. The off peak rates are low compared

with the peak periods. Customers usually request for a number of spots for their adverts. Table 2 shows request received by TV for Primetime News (19 h GMT). Customer 1 requested for

	Rates in $			
Category($i=\overline{1,4}$)	15 second	30 second	45 second	60 second
Prime times news (19h GMT)	215	375	562	750
News adjacencies	130	250	375	500
Break in news	135	244	362	525
Break in programme	91	160	220	360

two spots of adverts for fifteen sec each at prime time news. The cost of the two adverts is $ 430 (i.e., 215+215) as indicated in the value column. The weight of this advert is 30 sec.

Table 1: TV adverts rates

Table 2: Prime time news adverts−19:00 h GMT

Advertise No. j	Time in sec.	No.of spots requested	Category (weight)(w_j^1)	Cost $ (value) (v_j^1)
1	15	2	30	430
2	30	3	90	1125
3	45	1	45	562
4	15	1	15	214
5	30	3	90	1125
6	45	2	90	1124
7	60	1	60	750
8	30	2	60	750
9	45	2	90	1124

10	15	1	15	215
11	15	1	15	215
12	30	1	30	375
13	45	2	90	1124
14	15	2	30	430
15	30	2	60	1125
16	45	2	90	1124
17	30	3	90	1125
18	30	3	90	1125
19	45	2	90	1124
20	60	1	60	750
21	45	1	45	562
22	15	1	15	215
23	15	1	15	215
24	15	1	15	215
25	30	2	60	750
26	30	3	90	1125
27	15	2	30	430
28	60	1	60	750
29	30	3	90	1125
30	15	2	30	430

Additionally, customer number 5 requested three spots of 30 sec each, i.e. 90 sec (weight) with a cost of $1,125 (value). The total time available for adverts at the prime time news is 20 min (i.e., 1200 sec) but the total time requested is 1710 sec. Other customers opt for the News Adjacencies. This is 5 min before and after the prime time news at 19.00 h GMT. As shown in Table 3, the total time available is 10 min (600 sec) but the customers requested a total of 810 sec. Table 4 depicts the weights and the values for the adverts requested for the 22:30 news time. The total time available is 600 sec but the customers requested 720 sec.

Table 3: Adverts for news adjacencies −18:55 −19:00 and 20:00−20:05

Advertise No. j	Time requested (weight)(w_j^2)	Cost \$ (value)(v_j^2)
1	30	260
2	45	375
3	15	130
4	90	750
5	60	500
6	60	250
7	90	750
8	15	130
9	15	130

10	30	250
11	30	260
12	60	500
13	45	375
14	15	130
15	15	130
16	15	130
17	60	250
18	30	260
19	60	500
20	30	260

Table 4: Adverts for Break in News at 22:30 Hours GMT

Advertise No. j	Time requested (weight) (w_j^3)	Cost \$ (value)(v_j^3)
1	30	150
2	45	200
3	15	75
4	90	400
5	60	290
6	60	270

7	90	400
8	15	75
9	15	75
10	30	150
11	30	150
12	60	290
13	45	200
14	15	75
15	15	75
16	15	75
17	60	270
18	30	150
19	60	290
20	30	150

Table 5: Break in programme adverts for peak time on week days

Advertise No.	Time requested (weight)(w_j^4)	Cost $ (value)(v_j^4)
1	15	91
2	15	91
3	30	160
4	90	440
5	30	182
6	90	480

7	90	440
8	90	480
9	60	320
10	15	91
11	15	91
12	15	91
13	60	320
14	90	480
15	30	182
16	60	360
17	90	480
18	30	182

Finally the mathematical problem is formulated as knapsack optimization problem:

$$430x_{1,1} + 1125x_{2,1} + 562x_{3,1} + 214x_{4,1} + 1125x_{5,1} + 1124x_{6,1} + 750x_{7,1} + 750x_{8,1} + 1124x_{9,1} + 215x_{10,1} + 215x_{11,1} + 375x_{12,1} + 1124x_{13,1} + 430x_{14,1} + 1125x_{15,1} + 124x_{16,1} + 1125x_{17,1} + 1125x_{18,1} + 1124x_{19,1} + 750x_{20,1} + 562x_{21,1} + 215x_{22,1} + 215x_{23,1} + 215x_{24,1} + 750x_{251}, +1125x_{26,1} + 430x_{27,1} + 750x_{28,1} + 1125x_{29,1} + 430x_{30,1} + 260x_{1,2} +$$

$$375x_{2,2} + 130x_{3,2} + 750x_{4,2} + 500x_{5,2} + 250x_{6,2} + 750x_{7,2} + 130x_{8,2} + 130x_{9,2} + 250x_{10,2} + 260x_{11,2} + 500x_{12,2} + 375x_{13,2} + 130x_{14,2} + 130x_{15,2} + 130x_{16,2} + 250x_{17,2} + 260x_{18,2} + 500x_{19,2} + 260x_{20,2} + 150x_{1,3} + 200x_{2,3} + 75x_{3,3} + 400x_{4,3} + 290x_{5,3} + 270x_{6,3} + 400x_{7,3} + 75x_{8,3} + 75x_{9,3} + 150x_{10,3} + 150x_{11,3} + 290x_{12,3} + 200x_{13,3} + 75x_{14,3} + 75x_{15,3} + 75x_{16,3} + 270x_{17,3} + 150x_{18,3} + 290x_{19,3} + 150x_{20,3} + 91x_{1,4} + 91x_{2,4} + 160x_{3,4} + 440x_{4,4} + 182x_{5,4} + 480x_{6,4} + 440x_{7,4} + 480x_{8,4} + 320x_{9,4} + 91x_{10,4} + 91x_{11,4} + 91x_{12,4} + 320x_{13,4} + 480x_{14,4} + 182x_{15,4} + 360x_{16,4} + 480x_{17,4} + 182x_{18,4} \rightarrow \boldsymbol{max}$$

subject to

$$30x_{1,1} + 90x_{2,1} + 45x_{3,1} + 15x_{4,1} + 90x_{5,1} + 90x_{6,1} + 60x_{7,1} + 60x_{8,1} + 90x_{9,1} + 15x_{10,1} + 15x_{11,1} + 30x_{12,1} + 90x_{13,1} + 30x_{14,1} + 60x_{15,1} + 90x_{16,1} + 90x_{17,1} + 90x_{18,1} + 90x_{19,1} + 60x_{20,1} + 45x_{21,1} + 15x_{22,1} + 15x_{23,1} + 15x_{24,1} + 60x_{25,1} + 90x_{26,1} + 30x_{27,1} + 60x_{28,1} + 90x_{29,1} + 30x_{30,1} \leq 1200$$

$$30x_{1,2} + 45x_{2,2} + 15x_{3,2} + 750x_{4,2} + 90x_{5,2} + 60x_{6,2} + 60x_{7,2} + 90x_{8,2} + 15x_{9,2} + 30x_{10,2} + 30x_{11,2} + 60x_{12,2} + 45x_{13,2} + 15x_{14,2} + 15x_{15,2} + 15x_{16,2} + 60x_{17,2} + 30x_{18,2} + 60x_{19,2} + 30x_{20,2} \leq 600$$

$$30x_{1,3} + 45x_{2,3} + 15x_{3,3} + 90x_{4,3} + 60x_{5,3} + 60x_{6,3} + 90x_{7,3} + 15x_{8,3} + 15x_{9,3} + 30x_{10,3} + 30x_{11,3} + 60x_{12,3} + 45x_{13,3} + 15x_{14,3} + 15x_{15,3} + 15x_{16,,3} + 60x_{17,3} + 30x_{18,3} + 60x_{19,3} + 30x_{20,3} \leq 600$$

$$15x_{1,4} + 15x_{2,4} + 30x_{3,4} + 90x_{4,4} + 30x_{5,4} + 90x_{6,4} + 90x_{7,4} + 90x_{8,4} + 60x_{9,4} + 15x_{10,4} + 15x_{11,4} + 15x_{12,4} + 60x_{13,4} + 90x_{14,4} + 30x_{15,4} + 60x_{16,4} + 90x_{17,4} + 30x_{18,4} \leq 600$$

Where

1 if.jthadvertise of.ithcategory is selected ,$i = \overline{1,m}$, m=4and $n_1 = 30, n_2 = 20, n_3 = 20, n_4 = 18$.

Otherwise

o

3.Results of the analysis

By using software WINQSB we obtained theresults for the analysis of data from the Table 1 to 5 (Prime Time News, news adjacencies, Break in News and Break in programme)are shown below.

The optimal selection these adverts yielded \$ 26,659. From the Table 6, nineteen adverts were selected from the 30requested to give an optimal value of \$ 15,503. The selection for news adjacencies, the break in news, break in programme and a peak period yielded \$ 5,070, \$ 2,860, \$ 3,350, respectively.

Advert Category	No. of adverts requested	No. of adverts selected	Time availability (sec)	Optimal value $
Prime times news (19h GMT)	30	23	1200	15,379
News adjacencies	20	16	600	5,070
Break in news	20	17	600	2,860
Break in programme	18	13	600	3,350
Total				26,659

Table 6: (Provide self-explanatory caption)

Adverts numbers which selected inCategories :

Prime times news:
{1,2,4,5,8,10,11,12,14,15,17,18,19,20,22,23,24,25,27,28,29,30}
news adjacencies : {1,2,3,4,7,8,9,11,12,,13,14,15,16,18,19,20}
break in news : {1,2,3,5,6,8,9,11,12,13,14,15,16,17,18,19,20}
break in programme : {1,2,3,5,6,8,10,11,14,15,16,17,18}

4.Sensitivity analysis (The solution with post optimality)

4.1.Shadow prices

In most elementary treatment of linear programming, such as typically found in textbooks on Management Science and Operations Research, the dual variables of an LP are interpreted as marginal values of the right-hand side coefficients. As the latter often represent resources of limited supply, such marginal values have come to be known as shadow prices. They indicate how much additional units of the corresponding resources are worth. However, this equivalence between dual variables and shadow prices holds only under the assumption of non-degeneracy. Consider the following primal-dual pair of linear programs:

$\max c^T x$

Subject to

(p) $Ax \leq b$

$x \geq 0$

$$\min b^T y$$

Subject to

(D) $A^T y \geq c$

$y \geq 0$

Where A is mxn ,$c \in R^n$,$x \in R^n$,$b \in R^m$,$y \in R^m$

DEFINITION 1.

Denote the optimal objective value for (P), as a function of the right hand side, by

$v(b) = max\{c^T x | Ax \leq b; x \geq 0\}$

The set of feasible solutions and the set of optimal solutions to (P) by

$X = \{x \in R^n | Ax \leq b; x \geq 0\}$

$X^* = \{x \in X | c^T x = v(b)\}$

Respectively; and the set of feasible solutions and the set of optimal solutions to (D) by

$Y = \{y \in R^m | A^T y \geq c; y \geq 0\}$

$Y^* = \{y \in Y | b^T y = v(b)\}$

PROPOSITION 1 See[2]. $v(b)$is a non–decreasing, piecewise linear concave function.

DEFINITION 2. The set of sub gradients for a concave function $f: R^m \to R$is defined as

$$\partial f(b) = \{y \in R^m | f(b+u) \le f(b) + u^T y, for\ all\ u \in R^m\}$$

PROPOSITION 2 When v(b) is finite, $\partial v(b) = Y^*$

DEFINITION 3 The directional derivative of a function $f: R^m \to R$ at b in the direction of u is defined as

$$D_u f(b) = \lim_{t \to 0^+} \frac{f(b+tu) - f(b)}{t}$$

PROPOSITION 3 $D_u v(b) = min\{u^T y | y \in \partial v(b)\}$

DEFINITION 4. For (P), the buying (or positive) shadow price of constraint i is defined as

$$p_i^+ = D_{e(i)} v(b),$$

and the selling (or negative) shadow price of constraint i is defined as

$$p_i^- = -D_{-e(i)} v(b)$$

Where e(i) is the i th unit vector. In other words, the buying shadow price is the (instantaneous) rate of change in v(b)for an increase in bi and the selling shadow price is the negative of the (instantaneous) rate of change in v(b) for a decrease in b_i .

PROPOSITION 4

$$p_i^+ = \min\{y_i | y \in Y^*\}$$
$$p_i^- = max\{y_i | y \in Y^*\}$$

DEFINITION 5.

For any LP, the incremental shadow price $(p_i +\ )$ of constraint $i$is defined as the instantaneous rate of *improvement* in $v(b)$ with an increase in *bi*; the decremental shadow price $(p_i -)$ of constraint *i* is defined as the negative of the instantaneous rate of *improvement* in $v(b)$with a decrease in *bi*. In this sense, a negative incremental price or a positive decremental price is the rate of deterioration of the objective value as the right-hand side is changed. Note that whether an improvement actually involves an increase or a decrease in the objective value $v(b)$ depends on the direction of optimization. For example, when minimizing it is possible to incur an increase in $v(b)$ by decreasing the right-hand side of a less-than-or-equal-to constraint. The increase in $v(b)$ is a negative improvement. This gives a negative instantaneous rate of improvement with a decrease in *bi*. By definition, the decremental shadow price which is the negative of this rate comes out positive. Although it may appear somewhat cumbersome at first sight, this definition of shadow prices is necessary for consistency in the general case.

Table 7 interpretation of shadow prices

Minimizing

Type of constraints	Increase in $v(b)$	Decrease in $v(b)$
≤		$p_i + > 0$
≥		$p_i -> 0$
=		$p_i + > 0 \; or \; p_i -> 0$

Maximizing

Type of constraints	Increase in $v(b)$	Decrease in $v(b)$
≤		$p_i - < 0$
≥		$p_i + < 0$
=		$p_i + < 0$ or $p_i - < 0$

In our case study above the shadow prices for the constraint of the total time available for adverts at the prime time news is (12.5) , That is means if we increase the resource one unite , from 1200 sec. to 1201sec. the value of objective function improve (increase) by (12.5 $), And the shadow prices for the constraint of the total time available for adverts News adjacencies is (8.33), That is means if we increase the resource one unite , from 600 sec. to 601sec. the value of objective function improve (increase) by (8.33 $), And the shadow prices for the residual constraints(3and4) are 4.5 and 6.067 respectively.

4.2.Reduce costs

In linear programming, reduced cost, or opportunity cost, is the amount by which an objective function coefficient would have to improve (so increase for maximization problem, decrease for minimization problem) before it would be possible for a corresponding variable to assume a positive value in the optimal solution. It is the cost for increasing a variable by a small amount, i.e., the first derivative from a certain point on the polyhedron that constrains the problem. When the point is a vertex in the polyhedron, the variable with the most extreme cost, negatively for minimization and positively maximization, is sometimes referred to as the steepest edge.

It follows directly that for a minimization problem, any non-basic variables at their lower bounds with strictly negative reduced costs are eligible to enter that basis, while any basic variables must have a reduced cost that is exactly 0. For a maximization problem, the non-basic variables at their lower bounds that are eligible for entering the basis have a strictly positive reduced cost. For the case where x and y are optimal, the reduced costs can help explain why variables attain the value they do. For each variable, the corresponding sum of that stuff gives the reduced cost show which constraints force the variable up and down. For non-basic variables, the distance to zero gives the minimal change in the object coefficient to change the solution vector x.

For simplest if $(x_1, \ldots, x_n)$, and $(y_1, \ldots, y_n)$ is optimal for the dual, then we know:

$$c_1x_1 + \cdots + c_nx_n = b_1y_1 + \cdots + b_my_m$$

Left-hand side: Maximal revenue

Right-hand side: $\sum_i$ resources i(availibility of resource i) ×(revenue per unit of resource i)

In other words: Value of $y_i$at optimal is dual price of resource i

Away from optimality, we have

$$c_1x_1 + \cdots + c_nx_n < b_1y_1 + \cdots + b_my_m$$

Left-hand side: current (suboptimal) revenue

Right-hand side:

$\sum_i$ resources i(worth of resource i),Solution is not optimal because resources are not being fully utilized ,if $(x_1, \ldots, x_n)$is feasible (not necessarily optimal) for the primal, and $(y_1, \ldots, y_n)$ is the corresponding collection of dual values, then we know:

Current objective coefficient of x_j

= (Left-hand side of dual constraint$j$) - (Right-hand side)

= $(a_{1j}y_1 + \cdots + a_{mj}y_j) - c_j$

$c_j$is a measure of revenue per unit (of activity j) So $(a_{1j}y_1 + \cdots + a_{mj}y_j)$ is an imputed (implicit) cost per unit (of act. j) Also, y_j is imputed cost per unit of resource i in a unit of activity jIf objective coefficient ofx_j (= cost - revenue, = reduced cost) is strictly negative, then revenue >

cost, so it makes since to increase activity j — this is the pivoting process of the simplex method. Associated with each variable is a reduced cost value. However, the reduced cost value is only non-zero when the optimal value of a variable is zero. A somewhat intuitive way to think about the reduced cost variable is to think of it as indicating how much the cost of the activity represented by the variable must be reduced before any of that activity will be done, More precisely, the reduced cost value indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.

In our case study above the reduce cost (for example the non-basic variable x_{31} advert No.3 in category 1 , is (-0.5) that is means if that variable in optimal solution equal 1 , the value of objective function worse (decrease) by (0.5), And the non-basic variable x_{61} advert No.6 in category 1 , is (-1) that is means if that variable in optimal solution equal 1 , the value of objective function worse (decrease) by (1), And the non-basic variable x_{94} advert No.9 in category 4 , is (320) that is means if that variable in optimal solution equal 1 , the value of objective function improve (increase) by (320).

5. Conclusion

1. The amount of 26,659 obtained from adverts selected for the four categories of adverts, is far in excess of the arbitrary selection method used by GTV. The use of the software WINQSB is systematic and transparent as compared with the arbitrary method.

2. Higher returns can be achieved by GTV by the use of this software in their selection of adverts in the critical situations analyzed.

3. Marketing Managers/Programme Producers may benefit from the proposed approach for selecting adverts to guarantee maximized

profits for their TV stations. In an event where management may have to include certain adverts for national interest these could be isolated before selecting the others to compete for the limited time slots.

4.The concepts of marginal values and ranges have been explained using the optimal solution,the use of both shadow prices and reduced costs in sensitivity analysis has been demonstrated.

5. Sensitivity ranges have been introduced to provide validity ranges for the optimal objective function value and optimal basis.

6. Although there is some benefit in predicting the effect of changes in data, it has been shown that these indicators do have their limits.

References

1. S.k.Amposah, E.O.Oppong and E.Agyeman, optimal television adverts selection ,case study : Ghana television , research journal of information technology 3(1) : 49-54,2011.

2. James K.HO, Computing True Shadow Prices in Linear programming , INFORMATICA, 2000, vol.11No.4, 421-434.

3.G.B.Dantzig, Discrete Variable Extreme Problems, operations research , 5,266-277,1957.

4. A.Sinha and A.A.Zoltners, The multi-choice knapsack problem, Operations Research, 27,503-515,1979.

5. R.M.Nauss, The 0-1 knapsack problem with multi choice constraint, European journal of Operations Research, 2,125-131,1978.

6. G.Laport,The Vehicle Routing problem: An overview of exact and approximate algorithms , European journal of Operations Research 59,345-358,1992.

7.P.J.Kolesar , A branch and bound algorithm for knapsack problem , management science,13,723-735,1967.